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An Approach to the Analysis of Critical Elements of Transport and Logistics Networks Using Graph Theory

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ABSTRACT: The article's primary purpose is to develop methods for determining critical nodes, arcs, and routes of transport and logistics networks. One of the proposed algorithms resolves a problem in finding the critical elements of the transport network. The second gives solutions to make alternative paths based on this criticality of the network. These tools should be helpful for transport planners, especially for dangerous, oversized, and important goods. The basis of the developed method is the domination in graphs. The proposed analysis methodology is verified on the existing infrastructure in northeastern Poland.

1 INTRODUCTION

Given the current geopolitical situation and the previous crisis related to the COVID pandemic, it is reasonable to analyse transport and logistics networks in terms of their criticality. In the case of transport, this concept can be understood in various aspects.

For the proposed model, the criticality is analysed in the context of connectivity/reliability of vertices/edges, traffic flows in the transport network and the impact of events blocking their nodes/edges. The transport is one of the 8 Critical Infrastructures (CI) – defined by the Act of 26 April 2007 on crisis management (Journal of Laws 2007 No. 89 item 590). In the case of the European Union, the establishment of a critical infrastructure protection program was preceded by terrorist attacks on four city trains in Madrid in 2004, the subway in London in 2005 and an airplane flying from Great Britain to the USA in 2006. However, the functioning of this program made it possible to respond appropriately in 2010 to the paralysis of European air traffic caused by the volcanic eruption in Iceland. The effects of this event were felt not only on the old continent but all over the world.

The events mentioned above, and the consequences of their occurrence showed how much the national and global economies depend on the condition of transport (individual elements of the transport system), regardless of the branch to which these events are concerned.

The events of the last four years confirmed the justification for introducing a critical infrastructure protection policy. First, the COVID-19 pandemic showed how much societies depend on global transport. In turn, Russia's attack on Ukraine and the sanctions adopted by EU countries aimed at, among others, limiting oil and gas orders from Russia showed that it is necessary to be able to quickly make decisions about changing supply directions. Unfortunately, this is not an easy and quick process, so it is worth having prepared action scenarios in case of such situations. This is where various analytical, forecasting and decision-making models can be useful, making the transport system more resilient. The experience of the last several decades shows that

for the proper functioning of the economy and society, it is essential that the transport system (all its elements) is reliable and resistant to various unfavourable external factors. Therefore, you should work on methods that allow you to:
- finding weak links in the functioning of the transport system,
- to identify potential threats,
- to reduce or even eliminate these threats,
- to mitigate the effects of threats.

Threats to critical infrastructures may be of natural or anthropological origin; therefore, the methods of identifying threats depend on the source of their origin, and consequently, various methods are used to mitigate the effects of these threats. However, the most important thing seems to be finding weak links in the transport system. In the technical part, they will be related to means of transport or infrastructure and less often to legal regulations or economic aspects. For example, damage to a vehicle may result in blocking a section of linear infrastructure. However, this will be possible more often when there are bottlenecks in this infrastructure or increased traffic intensity in this section. When we find such elements of the transport system or, more specifically, the transport network, we can react in advance and counteract unfavourable crises. Thus, we come to the need to solve problems related to the reliability, resilience, vulnerability, or sensitivity of transport networks as the most crucial component in the transport system, allowing the implementation of its main tasks.

Thus, the article aims to establish critical paths (edges) and nodes (vertices), as well as their modelling, depending on the situation in the region. The tool can be used to determine new transport networks, considering crises that affect the determination of additional or other edges and dominant vertices. So far, the mathematical analysis models used have considered separately either the vertices or the edges in the graph representing the transport network. This article proposes a comprehensive approach to modelling - simultaneous consideration of vertices and edges - this is the novelty of this article. For the tested model, the criticality is analysed in the context of coherence/reliability of vertices/edges, traffic flows in the transport network, and the impact of events blocking their nodes/edges.

The structure of the paper is as follows. The Introduction section presents the main aspects of the considered problem. The state-of-the-art section shows the literature review. Further, the basic notations are defined. Finally, the theoretical methods and algorithms with real-world applications are introduced.

2 STATE OF THE ART

The introductory considerations state that the transport network is a "bloodstream" determining the region's development, directly proportional to the economic, development and military potential. The network is a chain of interconnected vessels whose opportunities and threats are replicated in other elements.

According to the literature ([23], [64]), the transport system is defined as a combination of elements such as means of transport, infrastructure, human resources and their interactions, which serve to generate demand for travel in a given area and to satisfy them by providing transport services. The above definition is very general but, at the same time, easy to adapt to various modes of transport. It allows the use of a structure of connections between the transport system elements adapted to the problem being solved. It was assumed that the best and most natural way of recording transport systems is a network representing the infrastructure of this system and making it possible to describe the flow of traffic flows occurring in it ([59], [64]). We should agree with this because the structure and traffic flows in the transport network best reflect the relationships between individual elements of the transport system and thus determine it. There are three main types of transport networks ([51], [64], [66]):
- All vertices of the same degree characterise regular transport networks. The regularity of the number of incident edges about the vertex indicates a high level of spatial organisation, as in the case of a road network [66].
- Small-world transport networks are characterised by dense connections between nodes in the close neighbourhood with a simultaneous small set of crucial connections with nodes in the further neighbourhood. A characteristic feature of this type of transport network is high sensitivity to failures around nodes with many close neighbours (sometimes called hubs) ([29], [66], [73]}.
- Scale-free transport networks are characterised by a hierarchical system with several vertices with many neighbours and a small number of vertices with a small set of neighbours (low vertex degree). This type of network is characterised by ease of evolution because adding new network nodes will prioritise connecting them to larger nodes ([9 - 10], [66]).

Due to the characteristics of transport infrastructure, analysing transport networks at the pre-design stage or assessing and improving them during operation is best carried out using mathematical modelling and optimization methods ([38],[45], [57-58], [77-78]).

The most common model for presenting the structure of transport networks is graphs. They are composed of two sets: V – vertices (nodes) and E – edges (arcs). They are denoted as G(V, E). One of the main applications of graph theory is the analysis of transport networks and systems ([32], [36], [38-40], [41], [44], [64], [57]). It was the attempt to solve the problem of bridges in Königsberg by the famous Swiss mathematician Leonhard Euler in 1736 that gave rise to graph theory and thus determined the meaning of the existence of this mathematical construction used to model and solve problems of transport networks ([3], [13-15], [30], [46], [60]) computer ([27], [29], [36], [66], [74]), and in recent years also social ([51], [73]).

The most crucial transport problems solved using graph theory include:
- the problem of the Chinese postman;
- travelling salesman problem;
- flows in networks;

– spanning tree.

Various ways of solving the Chinese postman problem can be found, among others, in the literature [32], [41]. In turn, various solutions to the travelling salesman problem are presented, among others, in [4], [5], [7], [37], [41], [55], [56], [57], [59], [64]. This problem has also been solved in terms of dynamic graphs, which take into account dynamic changes in the networks they represent, such as adding/removing edges or vertices ([2], [71], [76]).

Another problem of graph theory, which is essential in the analysis of any networks, is the search for the smallest or largest spanning tree [12], [36], [75] or the search for maximum flow in networks [35], [59].

Graph domination is essential in analysing transport networks ([19], [38 – 40], [44], [72]). It is used to solve problems such as:
– location - for example, the problem of minimizing the distance that a person must travel to reach the nearest facility offering services that are critical from the point of view of their life and health (e.g. hospitals, police or fire stations), assuming their constant number [43-44], [72];
– distance – for example, a problem in which the maximum distance to an object is fixed and the number of devices needed to serve all users should be minimized (e.g. base stations of a cellular network) [19], [43];
– related to the search for representative sets, for example, to monitor communication or electrical networks, as well as in geodetic measurements (one should find the smallest number of places where the surveyor must measure the height so that they cover the entire surveyed area) [11], [72];
– vulnerability of transport networks [17], [40], [77].

Moreover, by using graph theory to describe transport networks, Kansky proposed a series of indicators describing transport networks in the 1960s [69] at two levels of detail. The first refers to defining parameters or indicators concerning the entire network, while the second relates to the nodes. The primary assumption of their applicability is the fact that they provide methods allowing [69]:
– expressing the relationship between their values and the network structure,
– comparing transport networks with each other at specific moments in time,
– comparing the development of transport networks at different times.

In the 1980s, scientists dealing with networks, including transport and computer networks, concluded that searching for the shortest path in a network, the smallest spanning tree or the maximum flow in networks is insufficient. They began to pay attention to the connection problem between two vertices of the transport network. They thus turned their attention to network reliability issues, including transport networks. Currently, research on the reliability of transport networks is dominated by three main types:
– connectivity reliability – understood as the probability that network nodes will remain connected – a measure describing the topological structure of the transport network ([8], [13], [15-17], [23], [46], [49]);
– travel time reliability – understood as the probability that the journey between a given source and sink will take place within a specific period ([6], [14], [23], [50]);
– capacity reliability – understood as the probability that the capacity of the transport network is equal to or greater than the desired level when the capacity of the arc is random [24], [25], [50].

A concept closely related to the reliability of transport networks or, more broadly, transport systems is their sensitivity. Unfortunately, this concept is not clear. They can be understood in terms of reliability and risk in some situations [14], [31], [65]. The sensitivity of the transport network may also be expressed in terms of too low a level of services provided and related problems [16-17], [49], [52], [63], [68], as well as costs resulting from the unavailability of this network [18], [61]. Moreover, some studies point out that the sensitivity of the transport network is its low resistance to natural threats or threats from other sources [47-48]. In turn, other studies point to sensitivity related to rare but high risk [16], which correlates to the possibility of providing services and maintaining continuity of operation [34], [40], [76]. This approach corresponds to the concept of critical infrastructures [26], [28].

In particular, the last 25 years have made governments of many countries around the world aware that certain aspects of society's life are particularly exposed to the negative impact of natural factors, as well as those resulting from acts of terrorism or vandalism [52], [64]. Depending on the country, specific services, infrastructures, and, in a broader sense, sectors have been identified, which have been called critical because the lack of access to them causes significant perturbations and problems in the functioning of residents [20], [21], [22], [26], [28], [69]. Specific infrastructures or critical sectors allow for special attention to be paid to their safe and reliable operation [20-22], [40], [53]. For this reason, it is essential to have appropriate tools used when analyzing the functioning of these infrastructures [20-21], [31], [33], [40] and optimization [38], [53], [56].

A significant facilitation of the search for new, effective methods of analysis and optimization is the translation of the problem of transport network reliability into the language of the theory of systems and complex networks [1], [10], [51], [73]. Thanks to this, it is possible to use methods of analysis, modelling and optimization of the reliability and sensitivity of complex technical systems [20-21], [53], 62] about transport networks [13], [38-40].

3 BASIC NOTATIONS

In further research, we consider the simple, undirected (sometimes also weighted) graphs $\boldsymbol{G(V, E)}$ constituted by nodes (vertices) set V (or $V(G)$) and arcs (edges) set E (or $E(G)$). We adhere to the convention that $n=|V(G)|$ and $m = |E(G)|$. Let $u, v \in V(G)$, then the edge between these vertices in a simple graph is denoted as $\{u, v\}$, which in shortened form is often written as uv. According to the above definition, we consider unordered pairs. It should be noted that vertices are edge ends, i.e. they are adjacent to each other and, at the same time, incident to the edge they create.

The set of all vertices of the graph $G(V, E)$ adjacent to vertex v is called a neighbourhood and is denoted by $N_G(v)$ or $N(v)$. The set $N_G(v) \cup \{v\}$ is called the closed neighbourhood of a vertex v in the graph G and denoted by $N_G[v]$. For any subset of the set of vertices in graph G, i.e. $A \subseteq G$ the neighbourhood is defined as $N_G(A) = \bigcup_{v \in A} N_G(v)$. The degree of a vertex v in a graph G is the number of vertices belonging to the set $N_G(v)$ and is denoted as $deg_G(v) = |N_G(v)|$.

A line graph $L(G)$ (also called an adjoint, conjugate, covering, derivative, derived, edge, edge-to-vertex dual, interchange, representative) of a simple graph G is obtained by associating a vertex with each edge of the graph and connecting two vertices with an edge if and only if the corresponding edges of G have a vertex in common.

3.1 *Spanning trees*

A spanning tree $T_S(G)$ is a subset of a graph G, with all the vertices covered with the minimum possible number of edges. In other words, a spanning tree contains all the vertices of the graph and only those edges that do not form cycles in such a graph. Regarding the topology of such a tree, the smallest will mean the one whose set of edges is the smallest, while the largest will mean the most extensive such set of edges. When describing graph edges using numerical weights, the smallest spanning tree is the one for which the sum of the weights is the smallest. Similarly, the largest spanning tree is the one in which the sum of the edge weights is the largest.

The problem of finding a minimal spanning tree (for weighted graphs) uses Kruskal's algorithm as a way to resolve this problem. The short description of this algorithm is presented as algorithm 1. In case of simple undirected graph, the Kruskal's method [54] is proper under assumption that every edge weight is equal to 1. In this way, the minimal spanning tree is the tree with the minimum number of edges that constitute the spanning tree of graph G.

Algorithm 1 [Kruskal's algorithm]
1: **Input** (G, w_e) with fixed edge weighted function w_e;
2: **Output** Minimal spanning tree $T_S(G)$;
3: Find the least weight of the edges in the graph (if there is more than one, choose randomly). Mark as red colour;
4: Find the next smallest unlabeled (uncoloured) edge in the graph that is not marked in red and colour it;
5: Repeat step 4 until you reach every vertex in the graph (or until you have n-1 coloured edges)
6: The coloured edges form the desired minimum spanning tree.
7: return $T_S(G)$.

3.2 *Domination in graphs*

A dominating set for a graph G is a subset D of its vertices, so any vertex of G is either in D or has neighbours in D ([30], [35], [44]). The domination number $\gamma(G)$ is the number of vertices in the smallest dominating set for G. An edge-dominating set for graph G is a subset $D_E \subseteq E$ such that every edge not in D_E is adjacent to at least one edge in D_E. A minimum edge-dominating number equals the power of the smallest edge-dominating set [38-40], [70].

Based on this definition, algorithm 2 [40] is presented. An important assumption for him is that the input graph must be connected, simple, and non-empty.

Algorithm 2 [Minimal-vertex-dominating-set]
1: **Input** $G=(V,E)$.
2: **Output** Minimal vertex-dominating set of graph G.
3: Fix S:=0; all vertices are white.
4: **while** (white vertices exist) **do**
5: choose $v \in \{x \mid w(x) = max_u \in V\{w(u)\}\}$ and it has the maximal number of white neighbours.
6: $S := S \cup \{v\}$; vertices in S are coloured by black; their neighbours are grey.
7: **end while**.
8: **return** S.
9: **if** in S there are vertices, what dominates the same vertices **then**
10: delete a vertex from S, which dominates only vertices dominated by other vertices from S; return $D = S \setminus \{v\}$
11: **else**
12: $D=S$
13: **end if**
14: **return** D.

This algorithm is complemented by algorithm 3, which finds the value of the weighted domination number [45]. The pseudo-code is proposed below.

Algorithm 3 [Weighted-domination-number]
1: **Input** (G, wv) with fixed vertex weighted function;
2: **Output** weighted domination number $\gamma w(G)$ of graph (G, wv);
3: **for** i=1 to $|V|$ **do**
4: find dominating set of G, starting from $v_i \in V$, and save it as $D_i(v_i)$;
5: find total weight $W_i(v_i) = \sum_{v_i \in D_i} w_i(v_i)$ of dominating sets $D_i(v_i)$;
6: **end for**;
7: find $min\{W_i(v_i) : v_i \in V, i=1,\ldots,|V|\} \rightarrow \gamma_w(G)$;
8: **return** $\gamma_w(G)$.

The third algorithm introduced as the basis for further theoretical and practical results is a deterministic discrete-time model of fire spread on a graph G. Hartnell introduces this concept in [42], and call firefighting problem. This model of fire spread considered how firefighters can act to stop a fire outbreak. An outbreak of fire starts at a set of root vertices of G at time t=0. In response, firefighters defend f vertices at time t=1 [42]. The pseudo code of proposed algorithms for finding solution of this problem is presented as algorithm 4.

Algorithm 4 [Firefighting problem (or virus control)]
1: **Input** graph $G(V,E)$;
2: **Output** the set of vertices under fire and non-fire;
3: A fire breaks out at a vertex of a graph.
4: The firefighter (or defender) then chooses any vertex not yet on fire (or affected by the virus) to protect.
5: The fire and firefighter alternate moves on the graph due to the fire can no longer spread.
6: The firefighter has chosen a vertex; it is protected or safe from future fire moves.
7: After the firefighter's step, the fire spreads to all vertices adjacent to the ones on fire, except those protected.

4 THEORETICAL RESULTS

This part of the article presents new methods for finding critical elements in transport networks. The proposed algorithms are based on those shown in the previous section. This analytical tool can be used to establish critical paths (edges) and nodes (vertices) and their modelling, depending on the situation in the considered region. The criticality is analysed regarding the consistency/reliability of nodes (vertices) / arcs (edges), traffic flows in the transport network, and the impact of events blocking their nodes/arcs.

Before we propose new algorithms to identify the critical elements of transport networks, some preliminaries must be introduced to find the edge-dominating set. The algorithm 5 presents a way to find the minimal edge dominating set. It uses the concept of line graphs.

Algorithm 5 [Minimal-edge-dominating-set]
1: **Input** $G=(V,E)$.
2: **Output** Minimal edge-dominating set of graph G.
3: Define line graphs $L(G)$ of graph G;
4: use algorithm 1 for graph $L(G)$;
5: return $D_{L(G)}$¬
6: $D_E= D_{L(G)}$.
7: **return** D_E.

For simple, undirected graphs the edge-domination number is the cardinality of the minimal edge dominating set. The value of this number can be found based on the result obtained from algorithm 5. Because a transport network can be described by weighted graph, the method for finding an edge weighted domination number is proposed in algorithm 6.

Algorithm 6 [Edge-weighted-domination-number]
1: **Input** (G,w_e) with fixed edge weighted function;
2: **Output** edge weighted domination number $\gamma_{we}(G)$ of graph (G,w_e);
3: Define weighted line graph $L_w(G)$ of graph (G,w_e);
4: use algorithm 3 for graph $L_w(G)$;
5: return $\gamma_w(L_w(G))$;
6: $\gamma_{we}(G):= \gamma_w(L_w(G))$;
7: **return** $\gamma_{we}(G)$.

The next step is defined critical components of the transport or logistics networks. Referring here to the fireman's algorithm (algorithm 4) is necessary. A critical node (vertex) is the first node (vertex) in the graph G that is not subject to the fire expansion of the greedy algorithm 4, thus determining all vertices in the further branches of the graph. Analogically, the critical arc (edge) is the first free arc (edge) in the graph that is not subject to the fire expansion of the greedy algorithm 4, and thus determines all edges in the further part of the graph branches.

Now, the new approach to identify the critical elements of a transport network can be proposed. The solution for this problem is algorithm 7.

Algorithm 7 [Identifying the critical elements of the transport/logistic network]
1: **Input** $G(V,E)$ or $G(V,E,w_e)$ represents a transport/logistics network;
2: **Output** sets of critical vertices (V_S) and edges (E_S);
3: Find the minimum spanning tree of G - $T_S(G)$ (e.g. algorithm 1).
4: Find the dominating set of $T_S(G)$ (greedy algorithms: e.g. Algorithm 5 or 6).
5: Find the edge-dominating set of the $T_S(G)$ started in vertices from $D(T_S(G))$ (greedy algorithm and line graphs).
6: For these vertices and edges, use the Firefighter's algorithm (algorithm 4).
7: Find the strategic vertices and edges.

This algorithm uses algorithms 1-6 depending on what type of graph the transport network represents. It should be emphasized that it is a greedy algorithm. At the same time, the problem it solves is an NP-complete problem. Therefore, it should be noted that it is a tool supporting analysis and decision-making but requires some caution in interpreting its results. They may not always be optimal. However, based on this algorithm, we get the following possibilities. We can determine critical vertices that protect individual tree arcs to prevent the graph's further atrophy. If fire infects the graph, a „*fireman*" extinguishing fire at the vertices and edges guarantees the graph's stability. The first fire-free vertices and edges are called the CRITICAL VEREX and the CRITICAL EDGE – in these critical elements should be a logistical backup, including sufficient tools to prevent the problem from spreading (for example deals with congestion).

It is necessary to give a tool to reverse the "fire" (congestion) spreading in the network. It means finding alternatives for the "occupied" nodes and arcs or possibly creating a separate path to distribute the "fire". Therefore, we get new connections between system-inefficient nodes and those that can "take over" the additional "fire" (traffic flow). The proposed solution is given as the algorithm 8.

Algorithm 8 [Identifying the alternative paths]
1: **Input** $G(V,E)$ or $G(V,E,w_e)$ as the result of the algorithm 7;
2: **Output** alternative paths P_A;
3: Build an extended graph $G_E=(V_E,E_E)$ for analysed area;
4: find domination set in G of vertices occupied by "fire" with the highest degree $D_F(G)$;
5: **while** $((D_F(G)\neq\emptyset)$ and (not all vertices from $D_F(G)$ occupied by "fire" are connected)) **do**
6: select the highest degree vertex $w\in D_F(G)$;
7: **if** there is a connection in G_E between the vertex w and the vertex v occupied by the fireman in G **then**
8: connect them with via a new edge not occupied by the "fire" $(e\in E_E)$;
9: **else**
10: connect w with v through additional vertices from the set $V_E\neq V$ (if there is a relationship in the real connection network that allows the implementation of the transport task);
11: add edges e to set of an alternative paths P_A;
12: **end while**;
12: return P_A.

5 APPLICATIONS

This section shows possible applications of the methods introduced in section 4.

Let us consider a graph describing the model of the map of Poland, the northeastern region. The graph representing this region is on the figure 1. It is necessary to mention that there are several elements of the critical infrastructures in the considered region, e.g.: the waterpower plant in Włocławek, the power plant in Ostrołęka, the Dębe hydroelectric power plant (on the barrage damming water in Zegrzyńskie Lake), potential facilities in Pomerania: offshore and the power plant in Żarnowiec, as well as bridges: Zegrze, Serock, Pułtusk, Rozan.

Only the infrastructure that meets the requirements for transporting oversized and dangerous cargo was adopted for analytical purposes. Thus, we consider only a mixture of the railroad and roads, but we show graph representation for railways, roads, and inland waterways.

Figure 1 shows railway connections with weights corresponding to the speed achieved on a given route. The Pan-European Railway (TEN – T) route receives an additional weight of 5.

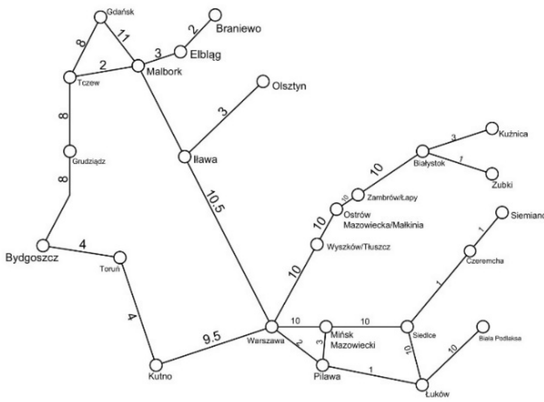

Figure 1. Selected railway connections. [own study]

The weights for figure 1 are defined according to table 1.

Table 1. Weights for graph represents the railway.

Velocity	Weight
to 80 km/h	1
81-100 km/h	2
101-120 km/h	3
121-140 km/h	4
141-160 km/h	5
161-200 km/h	6
Up to 200 km/h	7

Additionally, figure 2 presents road connections, with their weights corresponding to the road numbers. The Pan-European Railway (TEN – T) route receives an additional weight of 5.

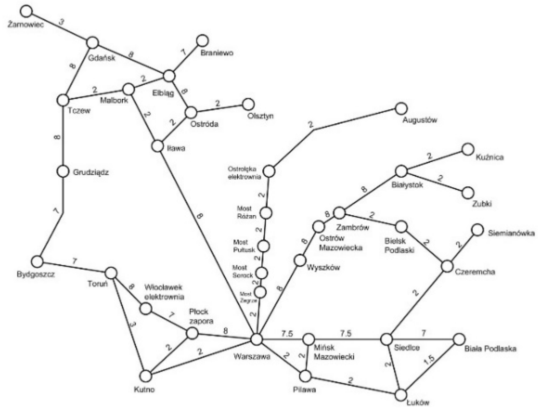

Figure 2. Selected roads connections. [own study]

The weights for figure 2 are defined according to table 2.

Table 2. Weights for graph represents the roads.

Road	Weight
Number with 1 digit	1
Number with 2 digits	2
Number with 3 digits	3

Finally, figure 3 presents inland connections, with their weights corresponding to the classes of inland waterways. The Pan-European Railway (TEN – T) route receives an additional weight of 5.

Figure 3. Selected inland waterways. [own study]

The weights for figure 3 are defined according to table 3.

Table 3. Weights for graph represents the inland waterways.

Class	1	2	3	4	5
Weight	1	2	3	4	5

In the general case, a graph representing all the modes of transport described above is presented in Figure 4.

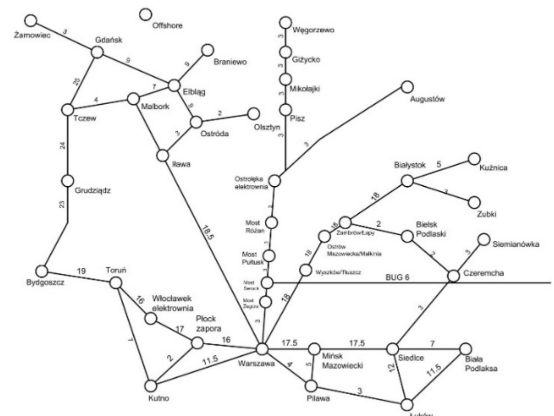

Figure 4. Summary of three modes of transport: rail, road, inland waterways. [own study]

According to assumption, we consider only roads and railways, as shown in figure 5.

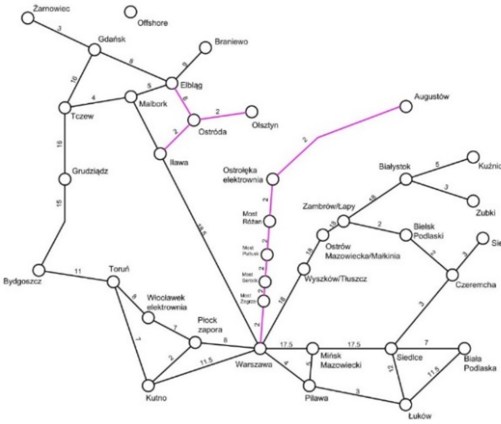

Figure 5. Summary of two modes of transport: rail, road. [own study]

For this graph (figure 5) we apply the algorithm 7 and get the minimal spanning tree (figure 6 – results according to step 3).

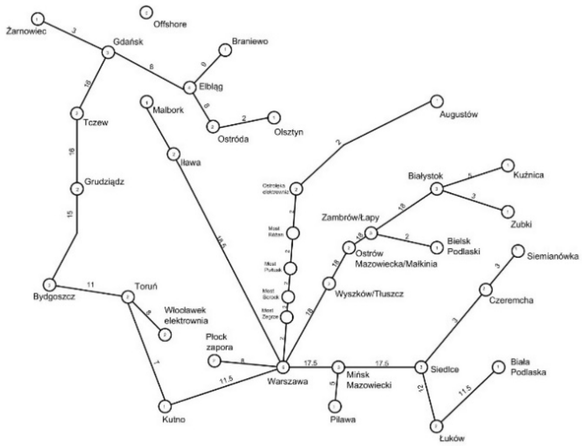

Figure 6. Spanning tree of graph represented considered transport network [own study]

Further we go to the step 4 of algorithm7. The results is in figure 7.

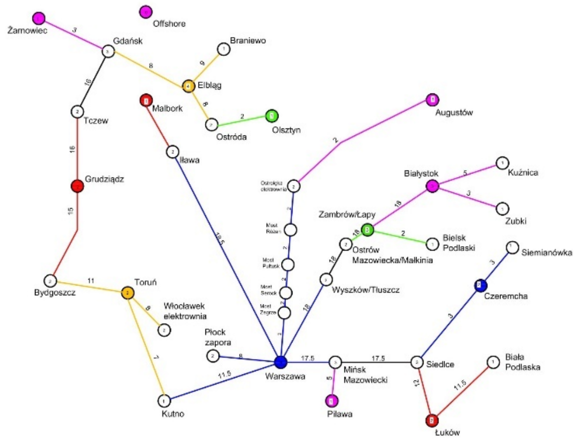

Figure 7. Domination set in spanning tree.

The resulting domination set of *Ts* is *D*={Żarnowiec, Czeremcha, Łuków, Elbląg, Olsztyn, Grudziądz, Malbork, Toruń, Warszawa, Augustów, Zambrów, Białystok, Pilawa, offshore}.

This set is the base for procedure of the firefighting algorithm. The fire is a congestion, and the "firefighter" is a traffic supervision. The steps of this are as follows.

Step 1 **FIRE** - vertices and edges coming from Warsaw: Mińsk Mazowiecki, Wyszków, Ostrołęka, Iława, Płock, Kutno.

Step 1a **FIREFIGHTER** - the occupied vertex with the highest degree should be searched for and secured (e.g. Mińsk Mazowiecki). From now on, both the top of Mińsk Mazowiecki and all edges are determined by the fireman and thus are not subject to fire expansion.

Step 2. **FIRE** - edges and vertices coming from Wyszków, Ostrołęka, Iława, Płock, and Kutno, thus occupying the vertices: Toruń, Augustów, Ostrów Mazowiecki, Malbork.

Step 2a **FIREFIGHTER** - the occupied vertex with the highest degree should be searched for and secured (e.g. Ostrów Mazowiecki). From now on, the fireman determines the top of Ostrów Mazowiecki and all edges; thus, they are not subject to fire expansion.

Step 3. **FIRE** takes the next edge from Toruń, thus occupying Bydgoszcz.

Step 3a **FIREFIGHTER** - the vertices and edges extending from the Grudziądz should be protected, thus securing the rest of the arc of the graph.

After this procedure, we get the results presented in Figure 8.

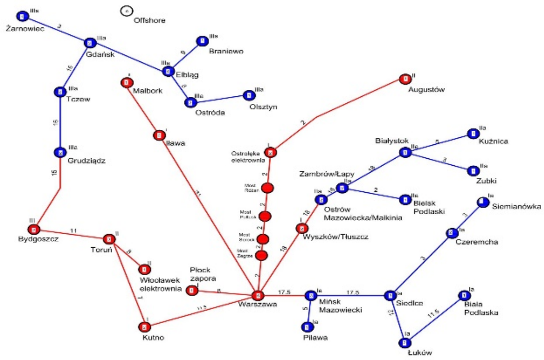

Figure 8. Resulting graph after step 6. [own study]

This way, we get the sets of critical nodes and arcs.

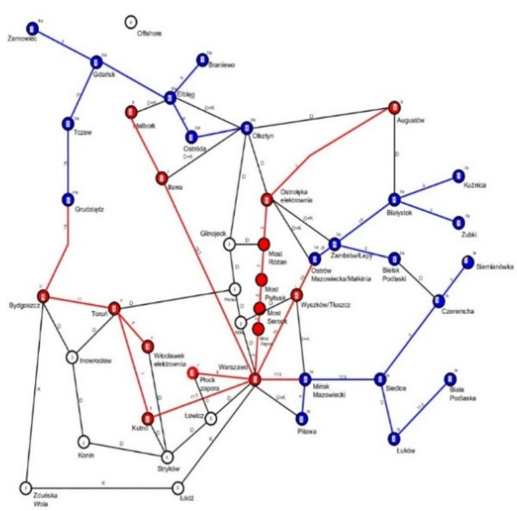

Figure 9. Alternative paths. [own study]

To reverse the congestion process (fire) in the network, it is necessary to look for an alternative to the "occupied" vertices/edges or create a separate path to distribute the congestion - following the theoretical results (Section 4, algorithm 8). The new connections between system-inefficient vertices and those that can "hijack" the additional traffic flow is presented (black in figure 9).

To summarize the practical part, it should be noted that:

1 Dominating set of nodes in spanning tree for two modes of transport: Warszawa, Zambrów/Łapy, Białystok, Pilawa, Łuków, Czeremcha, Augustów, Malbork, Olsztyn, Elbląg, Żarnowiec, Grudziądz, Toruń, offshore.
2 Critical nodes (with arcs): Mińsk Mazowiecki, Ostrów Mazowiecki, Grudziądz (with critical edges) – it should be secured (according to be critical for transport network).

6 CONCLUSIONS

The intended purpose of the article was achieved. New algorithms were proposed for analysing the criticality of transport and logistics networks. In particular, new algorithms were introduced for finding critical network elements (nodes and arcs) and searching for alternative routes for these elements.

The resulting model should be particularly subject to the Crisis Infrastructure Management Act because each edge is a bridge, the breaking of which will cut off strategic nodes in the operational management system.

The models can be a tool for determining the critical/strategic edges and vertices of the region, considering the optimization of transport reliability and costs.

Soon, the algorithms for strategic nodes and arcs will be proposed. Then, all models will be constituting the new decision support system for traffic flow planners and managers.

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